Association of Mathematics Teacher Educators

STANDARDS FOR MATHEMATICS TEACHER PREPARATION

PRELIMINARY DRAFT

FOR REVIEW AND FEEDBACK

Writing Team Members:

Nadine Bezuk, San Diego State University, chair
Jennifer Bay-Williams, University of Louisville, leadership team member
Douglas H. Clements, University of Denver, leadership team member
W. Gary Martin, Auburn University, leadership team member

Julia Aguirre, University of Washington, Tacoma
Tim Boerst, University of Michigan
Beth Burroughs, Montana State University
Ed Dickey, University of South Carolina
Elizabeth Hughes, University of Northern Iowa
DeAnne Huinker, University of Wisconsin, Milwaukee
Rochelle Gutiérrez, University of Illinois at Urbana-Champaign
Karen Karp, Johns Hopkins University
Jim Lewis, University of Nebraska
Travis Olson, University of Nevada Las Vegas
Randy Philipp, San Diego State University
Nicole Rigelman, Portland State University
Marilyn Strutchens, Auburn University
Kristin Umland, University of New Mexico
Dorothy White, University of Georgia

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AMTE's *Standards for Mathematics Teacher Preparation*

Preface

The future mathematical success of our nation's children is largely dependent on the teachers of mathematics they encounter in grades PK-12. Effective teachers of mathematics can expand students' options, not just in mathematics, but also in their career opportunities (Konstantopoulos, 2011). Teacher preparation programs in mathematics must ensure that all their candidates have the knowledge, skills, and dispositions to ensure all their students have access and opportunities to have meaningful experiences with mathematics that prepare them for their future needs in the workforce and as critical citizens, building on their existing cultural and linguistic resources.

The Association of Mathematics Teacher Educators (AMTE) is the largest professional organization devoted to the improvement of mathematics teacher education. AMTE includes more than 1,000 members supporting the preservice teacher education and professional development of teachers of mathematics at all levels from prekindergarten to grade 12 (PK-12). AMTE members include professors, researchers, teacher-leaders, school-based and district mathematics supervisors and coordinators, policy experts, graduate students, and others. This document is reflects AMTE's leadership in shaping the preparation of P-12 teachers of mathematics, including what *well-prepared beginning mathematics teachers* should know and be able to do upon completion of a program and what characteristics such programs must have to support their development.

Teachers make a difference for students, schools, and communities. There is strong evidence that well-prepared teachers are more effective at impacting mathematics learning, and therefore we see the critical need to ensure that all teachers of mathematics are well-prepared. Although what we know about the initial preparation of mathematics teachers has been limited, we have a growing research base that informs what teaching practices impact student learning and student experiences in mathematics classrooms. For example, research indicates that a focus only on teachers' behaviors has less positive effect than a focus on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject (Carpenter, Fennema, Peterson, & Carey, 1988; Kennedy, 1998; Kwong et al., 2007; Philipp et al., 2007).

A number of recent documents address various aspects of the initial preparation of mathematics teachers.[1] *The Mathematical Education of Teachers II* (METII) (Conference Board of Mathematical Sciences, 2012) addresses the mathematical content knowledge "well-started beginning teachers" should possess, and *The Statistical Education of Teachers (SET)* (American Statistical Association, 2015) addresses the statistical content knowledge that preservice teachers should learn. The National Council of Teachers of Mathematics' (NCTM) CAEP Standards (NCTM, 2012a,

[1] For the purposes of this document, mathematics teacher preparation includes preparation to teach statistics, following common practice. However, we recognize that statistics and statistics education, while related to mathematics and mathematics education, have their own identity.

NCTM, 2012b) describe what effective preservice teachers of secondary mathematics should know and be able to do, informing program reviews for middle and secondary mathematics programs. Additionally, organizations not specific to mathematics address mathematical teaching and learning. The Council for Exceptional Children (CEC) standards for beginning teachers requires that beginning professionals understand and use mathematics concepts in order to individualize learning for students (CEC, 2012). The National Association for the Education of Young Children (NAEYC) (2010) professional standards describe the importance of knowing mathematics and teaching it in ways that promotes sense-making and nurtures positive development. The Council for the Accreditation of Educator Preparation (CAEP) further elaborates on content expectations for preservice teachers, describing knowledge, skills, and dispositions of effective teachers (CAEP, 2015).

And, these standards on the initial preparation of teachers are influenced by standards for experienced teachers. The inTASC Model Core Teaching Standards (Council of Chief State School Officers, 2013) are used by states, school districts, professional organizations, and teacher education programs to support teachers. The National Board for Professional Teaching Standards (NBPTS) were developed to provide certification designed to retain and recognize accomplished teachers and include certifications for early and middle childhood generalist (early childhood and elementary), and early adolescence (middle school), and adolescence and young adulthood (high school) mathematics teachers. AMTE's *Standards for Elementary Mathematics Specialists* (2013) outlines "particular knowledge, skills, and dispositions" needed by elementary mathematics specialists who "teach and support others who teach mathematics at the elementary level" (p. iv).

While all of these standards inform mathematics teacher preparation, there is no single, comprehensive document addressing the initial preparation of mathematics teachers across PK-12. It is AMTE's vision that the standards provided in this document provide a clear, comprehensive vision for initial preparation of mathematics teachers, building on the standards briefly discussed above, and expanding on what beginning teachers of mathematics must know and be able to do, as well as the dispositions they must have, in order to increase equity, access, and opportunities for all students. Given the challenges that teachers of mathematics face within the current context in preparing their students for future success, it is imperative that mathematics teacher educators are guided by a well-articulated vision to help prepare teachers of mathematics to meet those challenges. This document takes up that charge.

Purpose

This document presents a set of comprehensive standards describing a national vision for the initial preparation of all teachers PK-12 who teach mathematics. That is, in addition to early childhood and elementary teachers who teach all disciplines, middle grade teachers, and secondary mathematics teachers, these standards are also directed towards special education teachers, teachers of English Language Learners, and all others who will have responsibility for aspects of student learning in mathematics.

These standards are intended to guide the improvement of individual teacher preparation programs, inform the accreditation process, and promote national dialogue and action related to mathematics teacher preparation. These standards are aspirational, advocating for mathematics teacher preparation practices that support candidates in becoming high quality teachers who effectively guide student learning in alignment with research and best practices. The standards are intended to both build on existing research about mathematics teacher preparation (and existing standards) and to inspire continued research in areas that are less well understood.

Audience

The audience for these standards includes the broad range of all those involved in mathematics teacher preparation, including faculty and others involved in the initial preparation of mathematics teachers; classroom teachers and other PK-12 school personnel who support student teachers and field placements; coordinators of mathematics teacher preparation programs; deans, provosts, and other program administrators who make decisions regarding content and funding of mathematics teacher preparation programs; CAEP, the largest accreditor of teacher education programs in the United States; state licensure or credentialing agencies/organizations; NCTM, the professional association responsible for setting standards for educator-preparation programs for preservice middle and high school mathematics; and other organizations, including specialized professional associations (e.g. NAEYC, CEC) and agencies focused on and involved in the preparation of mathematics teachers.

AMTE's *Standards for Mathematics Teacher Preparation*

Chapter 1: Introduction

Assumptions about Mathematics Teacher Preparation

As a community, mathematics teacher educators have begun to define, research, and refine the characteristics of effective teachers of mathematics, and in particular the professional proficiencies of a well-prepared beginning teacher of mathematics. This document describes a set of proficiencies for well-prepared beginners and for programs preparing mathematics teachers. Although these proficiencies are grounded in the available research, that research is often insufficient to describe in detail the knowledge, skills or dispositions that will enable a beginner to be highly effective in their first years of teaching. Hence, the standards presented in this document are intended to engage the mathematics teacher education community in continued research and discussion about what candidates must learn during their initial preparation.

Additionally, these standards are grounded in assumptions about mathematics teaching. The following five postulates describe the context in which we developed the standards presented in Chapters 2 and 3. These postulates reflect the emerging consensus of those involved in mathematics teacher preparation in response to the needs of both their teacher candidates and the students those candidates will teach.

Postulate #1: A commitment to ensuring the success of every learner is essential for every effective mathematics teacher, including well-prepared beginners, and for every program that prepares mathematics teachers. Over the past decades, the need for a central focus on issues related to equity in mathematics education has become clear in reflecting on the uneven performance of students by various demographic factors (AMTE, 2015; NCTM, 2000, 2014a 2014b). Although critical equity, diversity, and social justice issues need to be specifically addressed as standards, they must also be embedded within all the standards that are described. Addressing these issues solely within the context of an "equity standard" might inadvertently imply that these issues are not important to the other standards; conversely, if they are not directly addressed, their centrality to the mission of mathematics teacher preparation can get lost. Thus, we believe that equity must both be addressed in its own right and embedded within every standard. Every standard must be built on the premise that it applies to all learners, recognizing that equity requires acknowledging the particular context and the needs and capabilities of each learner rather than providing identical opportunities to all students.

Postulate #2: Teaching mathematics effectively requires career-long learning about teaching mathematics. Experienced teachers reflecting on their first year of teaching mathematics frequently describe how much more they can now accomplish given their current level of teaching competence and understanding of the mathematics they are teaching. Teachers improve through reflective experience and through intentional efforts to seek additional knowledge. They use that knowledge to build their understanding of the mathematics they teach

and to support their improvement in supporting students' learning of mathematics. This process must begin during their initial preparation with clear expectations for this transition point, and then continue throughout their careers. Knowing that teachers will complete programs without the expertise they will later develop focuses attention on priorities for teachers right from the start. Those become the knowledge, skills, and dispositions of a well-prepared beginner.

Postulate #3: Learning to teach mathematics requires a central focus on mathematics. Many times teaching is approached as a general craft that is independent of the content being taught. However, effective mathematics teaching requires not just general pedagogical skills, but also content-specific knowledge, skills, and dispositions. Mathematics teachers, at every level of instruction, need deep knowledge of the mathematics they teach and beyond, how students think about and learn mathematics, instructional approaches that support mathematical learning, and the societal context in which mathematics is taught to effectively support student learning of and positive dispositions toward mathematics.

Postulate #4: The preparation of teachers of mathematics is a shared responsibility of multiple stakeholders. Preparing teacher candidates to teach in ways that ensure all students learn important mathematics requires the concerted effort of everyone who holds a stake in their future success. Mathematics teacher educators, mathematicians, school administrators, classroom teachers, special education teachers, families and communities, policy-makers, and others in the educational system each play critical roles. When these groups send beginning mathematics teachers mixed messages about how mathematics is best taught and learned, those beginning teachers come to hold an incomplete, incoherent, and fragmented vision of how to enact an effective mathematics learning environment for their students. Successful mathematics teacher preparation requires a shared vision of mathematics learning outcomes for students, of effective mathematics learning environments, and of the kind of experiences that best support a mathematics teacher's continuing growth and development. Moreover, stakeholders must both feel included in the development of that vision and accountable for enacting that vision.

Postulate #5: Those involved in mathematics teacher preparation must be committed to improving their effectiveness in preparing future mathematics teachers. Mathematics teacher preparation programs should reflect research related to mathematics teacher preparation. However, the knowledge base for effective mathematics teacher preparation is far from complete. First, many important issues have only begun to be explored. For example, we know that when content-specific methods courses address equity issues (e.g., culturally-based algorithms (Civil, in press) or content-specific linguistic resources ELLs draw upon (Zahner, in press), prospective teachers better understand how to enact goals of equity while teaching mathematics, but much more can be learned about the types of experiences that best prepare a beginning mathematics teacher to support culturally and linguistically diverse students.

Second, it is often not clear how the research base that does exist might apply across the range of contexts in which mathematics teacher preparation occurs; in the United States there are hundreds of institutions, as well as online and school district programs, where a person can become a teacher of mathematics and none of them are exactly alike. Program structures differ

widely and the needs and backgrounds of their candidates vary. For example, students enrolled in a four-year undergraduate program in education may be exposed to many of the standards put forth in this document years before students who complete their teaching credential as part of a fifth year post-graduate credential program.

Thus, programs need to consider how existing research might apply to their context and how they can respond to issues not yet addressed by research. An emphasis on evaluating practices based on evidence will help to ensure effective decision-making. And as appropriate, the knowledge that is generated by particular programs should be contributed to the broader community of those involved in mathematics teacher preparation through publications, presentations at conferences, and other venues. Those able to conduct more formal research play an important role in exploring new directions for inquiry in mathematics teacher preparation.

The standards provided in the chapters that follow provide clear expectations based on the current knowledge base and national recommendations related to preparing effective teachers of mathematics and provide a framework on which individual programs can study their practices, and on which researchers can begin to look across programs to better understand and investigate critical aspects in the preparation of a well-prepared beginning teacher of mathematics.

Teacher Professional Continuum

As previously stated in Postulate 2, the development of teachers' content and teaching knowledge, skills, and dispositions develops over a career-long trajectory, as depicted in Figure 1. For example, the Interstate Teacher Assessment and Support Consortium (InTASC) developed learning progressions to describe "a coherent continuum of expectations for teachers from beginning through accomplished practice" (Council of Chief State School Officers, 2013, p. 6).

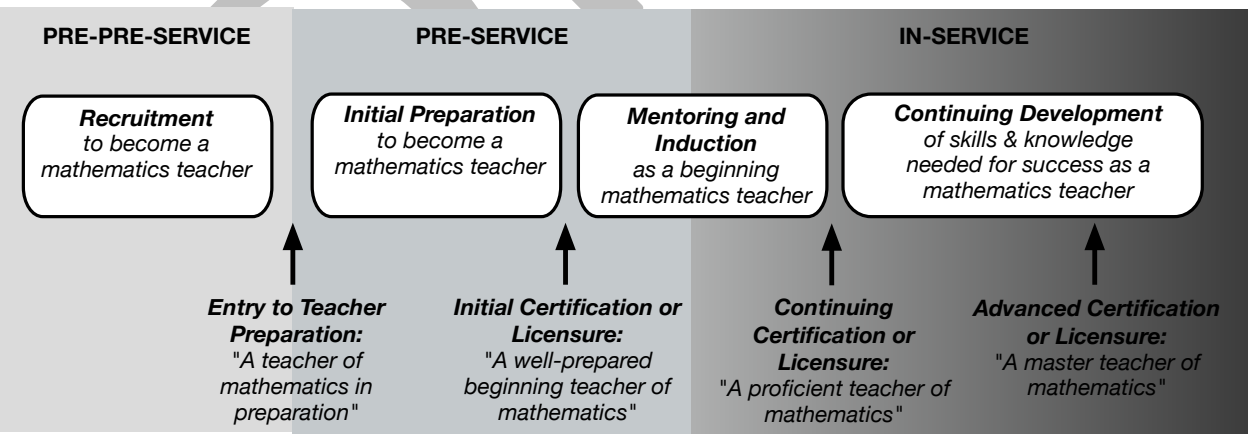

Figure 1. The teacher development continuum (adapted from Coble, 2012).

The standards in this document primarily address the initial two phases of the trajectory depicted in Figure 1, from recruitment of teacher candidates into a teacher preparation program to their

entry into the profession. However, the standards also set a strong foundation for the continuing growth and development of teachers of mathematics across the continuum.

A Well-Prepared Beginning Teacher of Mathematics

This document describes what a well-prepared beginning teacher of mathematics should know and be able to do, as well as productive dispositions they should develop. Well-prepared beginning mathematics teachers will be prepared to support the mathematical success of all students, and with proper support and incentives, they will continue to become more effective over the course of their careers.

In this chapter we set the overall framework including a set of postulates for the document. Chapter 2 provides standards for the professional knowledge, skills, and dispositions that well-prepared beginning mathematics teachers should possess related to content, teaching, learners and learning, and the social context of mathematics education.

Chapter 3 describes standards for mathematics teacher preparation programs designed to develop the knowledge, skills, and dispositions of their teacher candidates described in Chapter 2.

Chapter 4 will provide attention to assessment of candidates and programs.

Chapters 5 through 8 make specific recommendations about the preparation of mathematics teachers at different levels of instruction or grade bands, including Prekindergarten to grade 2, grades 3 through 5, grades 6 through 8, and grades 9 through 12.

A final chapter describes a general plan of action across stakeholder groups needed to enact these standards in mathematics teacher preparation programs across the nation.

Chapter 2: Candidate Knowledge, Skills, and Dispositions

Teaching is a complex enterprise, and teaching mathematics is particularly complex (e.g., Brousseau, 1997; Schoenfeld, 1988; Sowder, 2007). Thus, it is not surprising that initial preparation that focuses only on teachers' specific behaviors has less positive effect than a focus on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject (Ball & Forzani, 2011; Carpenter, Fennema, Peterson, & Carey, 1988; Kennedy, 1998; Kwong et al., 2007; Philipp et al., 2007). As described in *Council for the Accreditation of Education Preparation (CAEP) Accreditation Standards and Evidence: Aspirations for Educator Preparation*, teacher candidates must learn critical concepts and principle of their discipline and, by completion, are able to use discipline-specific practices flexibly" (2013, p. 3). This chapter describes the specific knowledge, skills, and dispositions that "well-prepared beginning teachers of mathematics" ("well-prepared beginners" for short), who are starting their careers after completion of a teacher preparation program, must be prepared to know and do, as well as productive dispositions they should hold. These competencies are developed in high-quality programs, characteristics of which are implicitly addressed throughout this chapter and then addressed explicitly in Chapter 3.

What Should Well-Prepared Beginning Teachers of Mathematics Know and Be Able to Do, and What Dispositions Should They Develop?

The guiding question for this chapter is, "Recognizing that learning to teach is an ongoing process over many years, what are reasonable expectations for the most important knowledge, skills, and dispositions that *beginning* teachers of mathematics must possess to be effective?" This is a difficult question to answer, as some aspects of teaching are not going to be well learned initially, even though they may be critically important to student learning. This is also a significant equity issue, as students with the greatest needs are often taught with teachers with the least experience both across and within schools (Boyd, Lankford, Loeb, & Wyckoff, 2005; Kalogrides, Loeb, & Béteille, 2013).

Mathematics teachers, from the very beginning of their careers, must understand the mathematical content knowledge for the age groups or grades that they may teach, along with content that comes before and after those age groups or grades—and in a different and deeper way than often presented in textbooks, curriculum documents, or standards. Such knowledge impacts their students' learning (e.g., Hill, Rowan, & Ball, 2005; Ma, 1999; National Mathematics Advisory Panel, 2008). Unfortunately, few teachers receive intensive, sustained, and content-focused professional development in mathematics (Birman et al., 2007; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007), making it paramount that teacher preparation programs provide substantial opportunities for candidates to learn the mathematics content they will be teaching, as well as more advanced mathematics that will inform the content they teach.

Well-prepared beginners must be ready to teach every child in their first classrooms. Although pedagogical skills develop over time, beginners must have an initial repertoire of effective and equitable teaching strategies--for example, in selecting tasks, orchestrating classroom discussions, building on prior knowledge, and connecting conceptual understanding and procedural fluency (NCTM, 2014b). All teachers, including well-prepared beginners, must hold positive dispositions about mathematics and mathematics learning, such as the notions that mathematics can and must be understood, and that all students can develop mathematical proficiency, along with a commitment to imbue their students with similar beliefs and dispositions.

Being able to teach effectively requires knowledge of learners and learning, both general pedagogical knowledge and knowledge specific to the learning and teaching of mathematics. Knowing the learner includes knowing about their background, interests, strengths, and personalities, as well as knowing how students *think* and *learn* related to the mathematics they will be teaching, including possible misconceptions and creative pathways they may take in learning (Ball & Forzani, 2011; Clements & Sarama, 2014; Sztajn, Confrey, Wilson, & Edgington, 2012). Well-prepared beginners must understand—at least at an initial level—how to assess the understandings and competencies of their students and use this knowledge to plan and modify instruction using research-based instructional strategies (e.g., Ball & Forzani, 2011; Shulman, 1986).

Mathematics teaching and learning is influenced by social, historical, and institutional contexts. Beginning teachers must be aware of learners social, cultural, and linguistic resources; know learners histories, and how power relationships affect students' mathematical identities, access and advancement in mathematics (e.g., Gutiérrez, 2013, Jackson & Wilson, 2012; Wager, 2014). For example, classroom dynamics and social interactions strongly influence students' emerging mathematical identities, which in turn impacts the students' learning opportunities.

Organization of this Chapter

This chapter includes four standards that describe the knowledge, skills, and dispositions that well-prepared beginners should acquire. The first standard, "Knowledge of Mathematics for Teaching," describes the content knowledge involved in the teaching of mathematics. The second, "Knowledge and Practices for Teaching Mathematics," describes research-based practices or strategies for effective mathematics teaching. The third, "Knowledge of Students as Learners of Mathematics," describes what teachers should know about their students' mathematical knowledge, skills, representations, and dispositions, for both individual students and groups of students. The final standard in this chapter, "Contexts of Mathematics Teaching and Learning," describes the knowledge and dispositions beginning teachers should have about the social, historical, and institutional contexts of mathematics impact teaching and learning.

Each standard includes a number of more-specific indicators for that standard, along with accompanying explanation. These standards and indicators apply for all well-prepared beginning

teachers of mathematics from prekindergarten through high school. While examples from a particular grade-band may be included to help explain a standard or indicator, Chapters 5 through 8 will provide grade-band-specific recommendations related to these standards.

Standard 2.1: Knowledge of Mathematics for Teaching

A well-prepared beginning teacher of mathematics possesses appropriate mathematical knowledge of and skill in mathematics needed for teaching. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions. They engage in appropriate mathematical practices and support their students in doing the same.

Indicators of this standard include:

2.1.1. Core Content Knowledge: *A well-prepared beginning teacher of mathematics has a solid and flexible knowledge of core mathematical concepts and procedures they will teach and the mathematical practices that their students will engage in.*

2.1.2. Analyzing Content-based Materials: *A well-prepared beginning teacher of mathematics can read, analyze, and interpret curriculum, assessment frameworks, and standards documents for their grade band.*

2.1.3. Mathematical Dispositions: *A well-prepared beginning teacher of mathematics understands that success in mathematics depends on a productive disposition towards the subject and hard work.*

2.1.4. Understanding the Mathematical Thinking of Others: *A well-prepared beginning teacher of mathematics can analyze different student approaches to mathematical work in the grade-level they teach and respond appropriately.*

2.1.5. The Nature of Mathematics: *A well-prepared beginning teacher of mathematics understands that mathematics is a human endeavor.*

A sample indicator with discussion:

2.1.1. Core Content Knowledge: *A well-prepared beginning teacher of mathematics has a solid and flexible knowledge of core mathematical concepts and procedures they will teach and the mathematical practices that their students will engage in.*

Well-prepared beginners can solve problems in more than one way, explain the meaning of key concepts, and explain the mathematical rationale underlying key procedures. For example, a well-prepared beginner for grades 3-5 should recognize that simplifying $3 \div \frac{1}{5}$ suggests the question, "How many fifths are in 3?" Using a visual diagram as in Fig. 1, as well as considering that there are five fifths in one whole, leads to the realization that the answer will be 3 x 5 or 15. This result will be generalized to recognize that dividing by any unit fraction is equivalent to

multiplying by the denominator. Thus, this procedure is built on a solid and flexible understanding of underlying mathematics. (See the grade-band chapters for additional examples.)

Figure 1. Fraction bar representation of the problem $3 \div \frac{1}{5}$.

Well-prepared beginners also understand key learning trajectories (c.f., http://ime.math.arizona.edu/progressions/) and can make important connections between the mathematics that comes in the grade-band before and the grade-band after the one in which they are prepared to teach. In the example above, well-prepared beginner for grades 3-5 will understand the importance of unit fractions in students' understanding of fractions, developed in earlier grades, and see how this result can be generalized to division by any proper or improper fraction, important content taught in the middle grades.

Their mathematical knowledge includes knowledge of mathematical processes and practices (NCTM, 2014b; National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010). They use mathematical language with care and precision. They can explain their mathematical thinking using grade-appropriate concepts, procedures, and language, including knowing grade-appropriate definitions and interpretations for key mathematical concepts. They can apply their mathematical knowledge to real-world situations appropriate for the grade levels and the students they will teach. They are able to effectively use representations and technological tools appropriate for the mathematics content they will teach. They can model doing mathematics as a sense-making activity that promotes perseverance, problem posing, and problem solving. In short, they model the kind of mathematical thinking that will be expected of their students.

Recommendations for the specific core content that well-prepared beginning teachers of mathematics at different levels should know are given in the *Mathematical Education of Teachers II* (Conference Board of Mathematical Sciences, 2012) and the *Statistical Education of Teachers* (American Statistical Association, 2015). Additional recommendations for the secondary level are given in the *NCTM CAEP Mathematics Content for Secondary Addendum to the NCTM CAEP Standards 2012* (revised 2015). Additional details are provided in the grade-band chapters.

Standard 2.2: Knowledge and Practices for Teaching Mathematics

A well-prepared beginning teacher of mathematics has begun building broad pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics for sense-making, understanding, and reasoning.

Indicators of this standard include:

2.2.1. Teaching for Access and Equity: A well-prepared beginning teacher of mathematics

structures learning opportunities and uses teaching practices that provide access, support, and challenge in learning rigorous mathematics to advance the learning of all students.

2.2.2 Planning for Effective Instruction: A well-prepared beginning teacher of mathematics plans for instruction by using knowledge of mathematics content, learning progressions, effective teaching practices, and the needs and prior experiences of each student.

2.2.3. Implementing Effective Instruction: The well-prepared beginning teacher of mathematics utilizes a core set of teaching practices that are effective for developing meaningful student learning of mathematics.

2.2.4 Analyzing Teaching Practice: A well-prepared beginning teacher of mathematics is developing as a reflective practitioner who elicits and uses evidence of student understanding to analyze the effect of his or her own teaching on students' learning of mathematics.

2.2.5. Enhancing Teaching through Collaboration with Colleagues, Families, and Community Members: A well-prepared beginning teacher of mathematics seeks collaboration with other education professionals, parents, caregivers, and community partners to provide the best mathematics learning opportunities for each and every student.

A sample indicator with discussion:

2.2.3. Implementing Effective Instruction: *The well-prepared beginning teacher of mathematics utilizes a core set of teaching practices that are effective for developing meaningful student learning of mathematics*.

Teachers must not only understand the mathematics they are expected to teach (Ball, Thames, & Phelps, 2008) and understand how students learn mathematics (National Research Council, 2005), they must be skilled in using content-focused instructional pedagogies that advance the mathematics learning of all students (Forzani, 2014). Well-prepared beginning teachers of mathematics have begun to develop skillful use of a core set of effective teaching practices to advance student learning of mathematics. They can articulate and demonstrate key components of this core set of mathematics teaching practices that includes: (1) establishing mathematics goals to focus learning, (2) implementing tasks that promote reasoning and problem solving, (3) using and connecting mathematical representations, (4) facilitate meaningful mathematical discourse, (5) posing purposeful questions, (6) building procedural fluency from conceptual understanding, (7) supporting productive struggle in learning mathematics, and (8) eliciting and using evidence of student thinking (NCTM, 2014b).

Effective mathematics instruction begins with clear expectations for student learning. Well-prepared beginners know the importance of using well-defined goals throughout a lesson to inform instructional decisions (NCTM, 2014b). For example, they know to discuss the mathematical purpose of a lesson with students and to reference the learning goals as appropriate during the lesson. They also know that a discussion of learning goals helps students stay focused and supports students' ability to better self-assess and monitor one's own learning

(Zimmerman, 2001).

Well-prepared beginning teachers know that effective instruction begins by engaging students in meaningful mathematical thinking and reasoning. For example, they know to introduce or launch mathematics lessons by drawing on students' prior knowledge and experiences, including culture, language, and interests, to ensure students are able to connect with the mathematical ideas and to build students' sense of identify and agency as mathematical learners (Aguirre & Zavala, 2012; AMTE, 2015; Turner et al., 2012). They also realize the importance of giving students time to struggle productively in exploring mathematical tasks without lowering the cognitive demand or taking over the thinking and reasoning of students (Stein, Grover, & Henningsen, 1996; Stigler & Hiebert, 2004), but also are ready to pose purposeful questions that will scaffold students through their struggles toward understanding.

Well-prepared beginning teachers understand the importance of supporting students in using and connecting varied mathematical representations. To that end, they have developed their own competence in using a large repertoire of mathematical representations including visual, physical, verbal, contextual, and symbolic forms to support student learning of specific mathematical ideas, concepts, and procedures (Lesh, Post, & Behr, 1987). For instance, they might give students squares tiles and grid paper to construct and decompose arrays when studying multiplication and the distributive property. It is apparent that they allocate substantial instructional time for students to use, discuss, and develop understanding of mathematical ideas by making connections among varied representations (NCTM, 2014b).

Effective instruction engages students as a classroom community centered on mathematical discourse and sense-making (Hufferd-Ackles, Fuson, & Sherin, 2004). Throughout a lesson, well-prepared beginners provide opportunities for students to talk with, respond to, and question one another in whole class discussions, as well as in partners or small groups. For example, they often begin lessons by having students work individually on a mathematical problem and then direct students to share their solution attempts with a partner. During this time they are carefully monitoring and supporting students as they make note of the varied approaches being used by the students. Then they carefully select and sequence some of the student approaches and solution strategies for whole-class analysis and discussion (Smith & Stein, 2011).

The well-prepared beginner consciously positions students as authors of ideas, who present, explain, and justify their reasoning using varied representations and tools. They are careful that this sharing does not just become show-and-tell, but engages all students in making sense of the mathematics by comparing, analyzing, and critiquing the reasoning of each other. Throughout the lesson, they elicit, build on, and honor student thinking, but are also very deliberate in making certain the key mathematical ideas remain prominent in whole class discussions.

Standard 2.3: Knowledge of Students as Learners of Mathematics

A well-prepared beginning teacher of mathematics seeks to understand the mathematical knowledge, skills, and dispositions of individual students in her/his classroom and also understands patterns and progressions of students' thinking within crucial mathematical

domains. A well-prepared beginner is committed to learning about the cultural, linguistic, and experiential resources that students bring with them to the classroom and begin to leverage those insights when planning and enacting instruction.

Indicators of this standard include:

2.3.1: Students' thinking about mathematics content: A well-prepared beginning teacher of mathematics understands students' mathematical thinking in at least one, and possibly more, well-defined content domain(s).

2.3.2: Students' engagement in mathematical practices: A well-prepared beginning teacher of mathematics is able to recognize core mathematical practices within what students' say, do and create across many mathematical content domains, with more in-depth and nuanced knowledge in particular content domains.

2.3.3: Students' mathematical dispositions: A well-prepared beginning teacher of mathematics knows key facets of students' mathematical dispositions and is sensitized to the ways in which dispositions may impact students' engagement in mathematics.

2.3.4: Capitalizing on student thinking and experiences: A well-prepared beginning teacher of mathematics will actively seek information about student thinking and experiences and will use what is known about student thinking and experiences to guide teaching.

A sample indicator with discussion:

2.3.1: Students' thinking about mathematics content: A well-prepared beginning teacher of mathematics understands students' mathematical thinking in at least one, and preferably more, well-defined content domain(s) (such as within number and operations). Such understanding includes knowledge about students' understandings, including pre-instructional or informal knowledge, common students' conceptions (and misconceptions), and, where the research knowledge exists, progressions of students' thinking within the content domain. Well-prepared beginning teachers are poised to develop understanding of student thinking in other content domains by applying what they know about learning from different sources (research, curriculum, interactions with other professions as well as with children and families) and what they know about facets of student thinking that are likely to be pivotal across content domains.

It is crucial for teachers to have a well-grounded sense of ways that students might think about that content (Jacobs, Lamb, & Philipp, 2010). Well-prepared beginners enter the classroom eager to learn about the unique ways that students may think about mathematics and with knowledge of common patterns of student thinking and available research-based learning progressions. This knowledge is useful in planning for instruction, but also provides a resource for sense-making of moment-to-moment interactions with students (Clements & Sarama, 2009). For beginners this knowledge is particularly important in content domains that are highly prevalent in the curriculum and those known to be crucial to support students' later mathematical success. For instance, in elementary grades, well-prepared beginners must have facility with a variety of ways in which students make sense of and approach number and operation. Just as important is the

career-long endeavor of learning about the ways that students think about and use mathematics in other domains. Well-prepared beginners are disposed to – and have skills that enable them to – learn in an ongoing way about students' ways of thinking in many mathematical domains.

Well-prepared beginners appreciate that students come to the classroom with unique mathematical perspectives and experiences. They know that the quality and focus of their teaching is impacted by the depth and detail of their insight into each student's mathematical thinking. They need to learn about students' informal ideas and invented approaches, as well as students' formal knowledge and understandings. Students' mathematical experiences and resources must also be part of what a well-prepared beginner appreciates, makes sense of, and builds upon to support mathematical learning. Well-prepared beginners are disposed to continually seek information about their students' thinking both because of the breadth of mathematics that they are teaching – knowing how students think about the span of mathematics that needs to be taught is a substantial undertaking – and because they know that student thinking is continually evolving. They know that learning about a student's thinking is enhanced by deliberately drawing on the insights of families, professional colleagues, and sources of information from beyond the classroom.

The knowledge of student thinking expected of a well-prepared beginning teacher of mathematics differs in quantity and quality from that of an accomplished experienced teacher who has committed herself to continuous professional learning. Quantitatively, accomplished experienced teachers understand the ways that students think about all of the mathematical content domains they teach. Qualitatively, accomplished teachers' knowledge of students' mathematical thinking have a level of depth, connectivity, and specificity that is not reasonable to expect of a well-prepared beginning teacher. Rather the broad and deep knowledge of the accomplished teacher is a product of professional learning that melds extensive teaching experiences, keen observation of a wide array of students, active engagement in professional development experiences, uptake of ideas from research and curriculum, and interaction with knowledgeable colleagues.

Standard 2.4: Social Contexts of Mathematics Teaching and Learning

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicators of this standard include:

2.4.1: Access and Advancement: Well-prepared beginning teachers of mathematics recognize the difference between access to and advancement in mathematics learning and work to make sure all students have both.

2.4.2: Mathematical Identities: *Well-prepared beginning teachers of mathematics recognize that their role is to cultivate positive mathematical identities of their students.*

2.4.3: Building on Students' Strengths: *Well-prepared beginning teachers of mathematics identify and implement practices that draw on students' mathematical, cultural, and linguistic resources/strengths and challenge policies and practices grounded in deficit-based thinking.*

2.4.4: Engaging Communities: *Well-prepared beginning teachers of mathematics recognize the limits of working alone and engage members in various communities to make informed decisions that benefit all students.*

2.4.5: Power and Privilege in the History of Mathematics Education: *Well-prepared beginning teachers of mathematics understand the roles of power, privilege, and oppression in the history of mathematics education and are equipped to question existing educational systems that produce inequitable learning experiences and outcomes for students..*

2.4.6: Ethical Practice for Advocacy: Well-prepared beginning teachers of mathematics are knowledgeable about and accountable for enacting ethical practices that allow them to advocate for themselves and to challenge the status quo on behalf of their students.

A sample indicator with discussion:

2.4.2: Mathematical Identities: *Well-prepared beginning teachers of mathematics recognize that their role is to cultivate positive mathematical identities of their students.*

"All mathematics teachers are identity workers," in that they contribute to the kinds of identities students are developing both in the classroom and outside of it" (Gutiérrez, 2013). More so than most subjects, doing mathematics can evoke strong emotions. Students feel they are good at mathematics or not good at mathematics. Students harbor perceptions about what someone who is good at math "looks like" and, even very young students can identify who in their classrooms are "good" at mathematics. Often times those identified (by themselves and others) as 'good at math' are those that are quick at performing algorithms. Research and standards provide a different description of what it means to be good at mathematics. For example, *Adding it Up* (NRC, 2001) describes a productive disposition as "the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy." Developing robust mathematical identities begins with a focus on robust goals for what is important to know and be able to do in mathematics and does not require students to assimilate behaviors of the dominant culture in order to be seen as legitimate participants in the classroom.

Classroom dynamics and social interactions have a strong influence on students' emerging mathematical identities. Students' active participation in mathematics classrooms, and how their peers and the teacher listen to and respect their ideas, has an impact on students developing mathematical identities (Aguirre, Mayfield, Ingram & Martin, 2013; Martin, 2009). Power dynamics can be seen in mathematics classrooms as students tune out someone who they don't think will share a good strategy, or as students working in groups do not take up the ideas of some of their group members. In diverse classrooms, issues of power and privilege arise as students from historically dominant groups sit side by side with students from historically

marginalized groups (Esmonde & Langer-Osuna, 2013; Turner, Dominguez, Maldonado, & Empson, 2013). Students' knowledge develops alongside of their mathematical identity, and the developing identity is influenced by the social and power dynamics in the classroom. In addition, stereotypes and a focus on achievement gaps can impede the development of robust mathematical identities, while leading to deficit-based thinking (Gutiérrez, 2008; McGee & Martin, 2011). Well-prepared beginners understand that knowledge, power, and identity are interwoven and emerge within a student's experience, in particular as they engage in classroom discussions and other discourse communities.

Well-prepared beginners know how to question and alter practices that privilege some individuals and/or exclude others. Some of these practices have long-standing history in U.S. classrooms. For example, "board races" and timed tests privilege those who are fast, excluding those who use more processing time, while also communicating that those who are fast are good at mathematics and that not everyone can be good at mathematics. Questioning patterns in classrooms, for example, asking higher level questions to students perceived by the teacher as more capable, though perhaps unintentional, can negatively impact the development of robust mathematical identities. In summary, well-prepared beginners view their planning, teaching, and assessment as "identity-in-the-making" (Gutiérrez, 2013), resisting explanations that position the student as inferior or on the margins of the classroom culture, and instead focusing on how to better support students growth and success in the mathematics classroom in order to ensure the student develops a robust mathematical identity.

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